Color conductivity and ladder summation in hot QCD

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The color conductivity is computed at leading logarithmic order using a Kubo formula. We show how to sum an infinite series of planar ladder diagrams, assuming some approximations based on the dominance of soft scattering processes between hard particles in the plasma. The result agrees with the one obtained previously from a kinetical approach.

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I. INTRODUCTION

Color conductivity has turned out to be a significant quantity, for it enters directly into the theory governing the dynamics of long wavelength excitations in the QCD plasma. This was not obvious *a priori*, because a hydrodynamical description of the plasma usually involves transport coefficients associated with relaxation processes of momentum degrees of freedom for which the collision frequency is $\nu_p \sim g^4 T \log(1/g)$ [1], while the collision frequency for color relaxation is $\nu_c \sim g^2 T \log(1/g)$ [2]. However, a major achievement in understanding the dynamics of very soft excitations in the QCD plasma has been Bödeker's finding of an effective theory described by a Langevin equation, in which thermal noise is parametrized by the static color conductivity [3,4]. Although he did not use explicitly the concept of color conductivity, the leading logarithmic order result was implicit in his computation. As an intermediate step towards Bödeker's theory, Arnold, Son, and Yaffe [5] have proposed a Boltzmann equation from which color conductivity can be obtained. Along the way, some points of previous studies of color conductivity [2,6] have been clarified. Later, a rigorous derivation of the Boltzmann equation, starting from the quantum field equations, was given in Ref. [7]. The Boltzmann equation leading to Bödeker's theory (and color conductivity) can also be obtained starting from classical transport theory [8] or from the kinetic formulation of the hard thermal loop (HTL) effective theory [9]. Furthermore, a recent diagrammatic analysis by Bödeker [10] of the polarization tensor beyond the hard thermal loop approximation has also displayed the form of the Boltzmann equation. Some calculations of the color conductivity at next-to-leading log order have appeared over the last year [11,12].

These studies of color conductivity at leading-log order have used a kinetic approach. Other transport coefficients in hot gauge theories have also been studied over the last years [1,13,14,6] using this approach. But there is an alternative way to compute transport coefficients, namely, through the use of Kubo formulas. However, this approach poses a difficulty: a summation of an infinite series of diagrams is required. Only for a scalar theory has this summation been carried out explicitly [15]. The higher order contributions were shown to come from ladder diagrams. In addition, the equivalence under certain conditions of a kinetic equation to the field theoretical computation was established [15,16]. For the case of gauge theories it has been suggested that this kind of diagrams contributes at leading order [5]. However, because of the difficulty in carrying out these computations, this approach has not been pursued further.

The purpose of this paper is to show how the ladder diagrams can be explicitly summed at leading-log order within the imaginary time formalism of thermal field theory. We will work at high temperature so the coupling constant is small and all zero temperature masses can be neglected.

II. COLOR CONDUCTIVITY

The static color conductivity associated with color flow may be defined by the constitutive relation $\langle j_a^i \rangle = \sigma_{ab}^{ij} E_b^j$, where E_b^j are the components of an external, constant chromoelectric field and $\langle j_a^i \rangle$ is the ensemble average of the spatial part of the conserved color current density. Using linear response theory one may express color conductivity in terms of the low frequency, zero momentum limit of the retarded correlation function between color currents [17]. The corresponding formula is

$$\sigma_{ij}^{ab} = -\frac{\partial}{\partial q^0} \mathrm{Im}\, \Pi_{ij}^{Rab}(q^0, \mathbf{q}=0)\big|_{q^0=0}. \tag{1}$$

Because of isotropy, the dependence on the spatial and color indexes is very simple, $\sigma_{ij}^{ab} = \delta_{ij}\delta^{ab}\sigma_c$. The most efficient way to compute σ_c from the above Kubo formula is to exploit the Lehmann representation, which provides a direct connection via analytic continuation between the retarded Green's function and its counterpart in the imaginary time formalism, $\Pi^R(q^0,\mathbf{0}) = \Pi(\nu_q = -iq^0 + 0^+, \mathbf{0})$.

The dominant contribution would come from the diagrams in Fig. 1 with all other diagrams of higher order in the loop expansion being subleading *a priori*. However, all ladder diagrams of Fig. 2 will contribute at the same order, as will be shown in our explicit computation. This same kind of diagrams has been shown by Jeon [15] to give contributions of the same order in λ as the one-loop graph for the viscosity in $\lambda\phi^4$ theory. In the context of gauge theories, it has been suggested [5] that the same kind of diagrams gives the leading order contribution. This can be understood in the follow-

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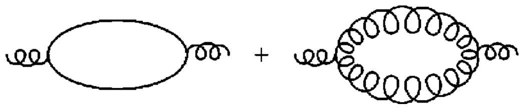

FIG. 1. One-loop contribution to gluon polarization tensor.

ing way. In the limit of zero spatial momentum and small frequencies, there are going to be pairs of propagators which will carry the same momentum, and this would lead to an on-shell singularity for each product of propagators with nearly the same momenta. The solution is to include in the propagators of the side rails their thermal widths Γ. Thus, for each pair of propagators, the singularity is replaced with a term $1/\Gamma$. Hence, for a general ladder graph with n rungs we will have $g^2(1/\Gamma)^{n+1}(g^2)^n$ where the first g^2 comes from the external vertices and each rung introduces a factor g^2 [15,18]. For gauge theories, the thermal widths for hard momenta are proportional to $g^2T\log(1/g)$, where the logarithm arises from the infrared behavior of the theory. As we shall see, a similar logarithm is produced by each integration over the small momentum transfer in the rung. So, both the one-loop and the ladder graphs are proportional to $g^2T^2q^0/\Gamma$. Other ladder diagrams with two gluons lines in the rung are subleading because the four gluon vertex has one more power of the coupling constant than the three gluon vertex.

Although we lack a rigorous power-counting scheme to dismiss all other possible higher order diagrams in the loop expansion, we do not expect them to contribute at leading order, since we are not aware of any other way in which the coupling constants of each new vertex could be compensated.

We now turn to the main point in this work, that is, how to sum the ladder diagrams of Fig. 2. First, we study the case when the side rails of the ladder are quark propagators.

A. The quark contribution

To include the thermal width in the propagators of the side rails, we modify the quark propagator by changing the delta functions of the spectral density for the free propagator by Lorentzians of width $2\gamma_f$

$$\rho(\omega,\mathbf{p})=S_+(\hat{\mathbf{p}})\frac{2\gamma_f}{(\omega-|\mathbf{p}|)^2+\gamma_f^2}+S_-(\hat{p})\frac{2\gamma_f}{(\omega+|\mathbf{p}|)^2+\gamma_f^2}, \quad (2)$$

where $S_\pm(\hat{\mathbf{p}})=(\gamma^0\mp\gamma\cdot\hat{\mathbf{p}})/2$ and γ_f is the fermion damping rate at hard momentum [19]

$$\gamma_f=\frac{N_c^2-1}{2N_c}\frac{g^2}{4\pi}T\ln(1/g). \quad (3)$$

From this spectral function the fermion propagator follows, as

FIG. 2. Ladder diagrams contributing at leading order to gluon polarization tensor. The gluon line with the blob denotes the HTL resummed propagator.

$$S(\omega_n,\mathbf{p})=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\rho(\omega,\mathbf{p})}{i\omega_n-\omega}=S_+(\hat{\mathbf{p}})\Delta_+(p)+S_-(\hat{\mathbf{p}})\Delta_-(p)$$
$$=\frac{S_+(\hat{\mathbf{p}})}{i\omega_n-|\mathbf{p}|+i\gamma_f\mathrm{sgn}(\omega_n)}+\frac{S_-(\hat{\mathbf{p}})}{i\omega_n+|\mathbf{p}|+i\gamma_f\mathrm{sgn}(\omega_n)}. \quad (4)$$

To sum the ladder contributions we write the following equation for an effective vertex

$$\Gamma^{i,a}(p+q,p)=\Gamma_0^{i,a}+g^2T\sum_n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\gamma^\alpha t^b \times S(k+p+q)\Gamma^{i,a}(k+p+q,k+p)\times S(k+p)\gamma^\beta t^b D_{\alpha\beta}(k), \quad (5)$$

where the zero components of the momenta are i times the corresponding Matsubara frequencies. $\Gamma_0^{i,a}=\gamma^i t^a$ is the tree level interaction vertex and t^a are the $SU(N_c)$ generators in the fundamental representation, with $\mathrm{tr}(t^at^b)=\delta^{ab}/2$. The gluon propagator $D_{\alpha\beta}$ is the HTL resummed propagator whose inclusion will be shortly justified. This equation is shown diagrammatically in Fig. 3. To solve it exactly is beyond all hope, so we will try to extract the dominant behavior by making some approximations.

The inclusion of the HTL propagator in the rung is based on the fact that the interaction of two particles must be modified due to many body effects for small momentum transfer $k\ll T$. It has the form

$$D_{\alpha\beta}(k)=\mathcal{P}^T_{\alpha\beta}(\hat{\mathbf{k}})\frac{1}{\omega_n^2+|\mathbf{k}|^2+\Pi_T(k)}+\delta_{4\alpha}\delta_{4\beta}\frac{1}{|\mathbf{k}|^2+\Pi_L(k)}, \quad (6)$$

where $\mathcal{P}^T_{ij}(\hat{\mathbf{k}})=\delta_{ij}-\hat{k}_i\hat{k}_j$, $\mathcal{P}^T_{4\alpha}=\mathcal{P}^T_{\alpha 4}=0$. In order to perform the Matsubara sum in Eq. (5), it is convenient to introduce the spectral representation of this propagator

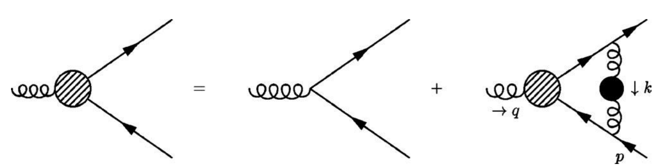

FIG. 3. Equation for the effective vertex which sums the quark ladder contributions.

$$D_{\alpha\beta}(\omega_n,\mathbf{k})=\frac{1}{\mathbf{k}^2}\delta_{4\alpha}\delta_{4\beta}+\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{i\omega_n-\omega}\times[-\rho_T(\omega,\mathbf{k})\mathcal{P}^T_{\alpha\beta}+\rho_L(\omega,\mathbf{k})\delta_{4\alpha}\delta_{4\beta}], \tag{7}$$

and a double spectral representation for the vertex, whose general form parametrized by two functions $F^{ia}_{1,2}(\omega_1,\omega_2)$ is given by

$$\Gamma^{i,a}(p+q,p)=\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\left[\frac{F_1^{ia}(\omega_1,\omega_2)}{(p^0-\omega_1)(q^0-\omega_2)}+\frac{F_2^{ia}(\omega_1,\omega_2)}{(p^0+q^0-\omega_1)(q^0-\omega_2)}\right]. \tag{8}$$

Inserting the spectral representations into Eq. (5), this reduces in the high temperature limit $\omega\ll T$ to

$$\Gamma^{i,a}(\omega_p+\nu_q,\omega_p;\mathbf{p})=\Gamma_0^{i,a}-g^2T\int\frac{d\omega}{2\pi}\frac{d^3\mathbf{k}}{(2\pi)^3}\gamma^\alpha t^b S\times(\omega_p+\nu_q,\mathbf{k}+\mathbf{p})\Gamma^{i,a}\times(\omega_p+\nu_q,\omega_p;\mathbf{k}+\mathbf{p})\times S(\omega_p,\mathbf{k}+\mathbf{p})\gamma^\beta t^b\frac{\rho_{\alpha\beta}(\omega,\mathbf{k})}{\omega}, \tag{9}$$

where the factor T/ω comes from the term containing the Bose-Einstein distribution $N(\omega)$ produced in the summation and ω_p,ν_q denote the fermionic and bosonic Matsubara frequencies of the external lines. Thus, the summation has been traded for an integration over ω.

To proceed further in the integration, let us recall the relevant kinematical domain from which arises the logarithmic sensitivity of the hard damping rates to the magnetic mass of order g^2T. As we shall see, the leading-log order in the color conductivity results from the momentum transfer in the same domain.

As is well known, the transverse part of the interaction is dynamically screened due to Landau damping of the magnetic virtual gluons in the regime $|\omega|\leqslant|\mathbf{k}|$, and static Debye screening cuts off effectively the small angle divergences of the longitudinal interaction. In terms of the gluon spectral density, the (k,ω) integral expressing the hard thermal damping rate (3) is [19–21] given by

$$\gamma_f=\frac{N_c^2-1}{2N_c}\frac{g^2T}{4\pi}\int_{\Lambda_{\min}}^{\Lambda_{\max}}dk\,k\int_{-k}^{k}\frac{d\omega}{2\pi\omega}\times\left[\rho_L(\omega,k)+\left(1-\frac{\omega^2}{k^2}\right)\rho_T(\omega,k)\right], \tag{10}$$

where the lower cutoff $\Lambda_{\min}$, which is taken of the same order as the hypothetical magnetic mass $\sim g^2T$, regularizes the infrared divergence of the integrand. The leading-log contribution to the damping rate arises from the exchange of quasistatic magnetic gluons in the $\omega\ll k$ limit. The reason for this is that the denominator of the integrand $k\rho_T/\omega$ vanishes in the static limit for $k=0$, while the denominator of $k\rho_L/\omega$ remains finite in this limit. In that domain, we can use the approximate expressions for the spectral densities

$$\rho_T(\omega,\mathbf{k})=\frac{8\pi m_D^2\omega k}{16k^6+\pi^2m^4\omega^2}, \tag{11}$$

$$\rho_L(\omega,\mathbf{k})=\frac{\pi m_D^2\omega}{k(k^2+m_D^2)^2}, \tag{12}$$

where $m_D^2=(N_f+2N_c)g^2T^2/6$ is the Debye mass. Inserting them into Eq. (10) produces a dominant logarithmic term

$$\gamma_f=\frac{N_c^2-1}{2N_c}\frac{g^2T}{2\pi^2}\int_{\Lambda_{\min}}^{\Lambda_{\max}}\frac{dk}{k}\arctan\left(\frac{\pi m_D^2}{4k^2}\right)\approx\frac{N_c^2-1}{2N_c}\frac{g^2T}{4\pi}\int_{\Lambda_{\min}}^{\Lambda_{\max}}\frac{dk}{k} \tag{13}$$

coming entirely from the transverse part, together with a finite term coming from the longitudinal part which is of order g^2T. Here, we only need the most singular logarithmic term, but for a computation of other transport coefficients, all contributions are possibly required. The upper cutoff $\Lambda_{\max}$ must be chosen of order gT because when $k\gg gT$, the contribution from hard momentum transfer $k\sim T$ is computed using a non resummed gluon propagator. Thus the contribution to the damping rate from this domain is proportional to

$$g^4T^3\int_{\Lambda_{\max}}^{\infty}\frac{dk}{k^3}. \tag{14}$$

If the upper cutoff were chosen of order T in Eq. (13), one would make the mistake of relying on an integrand which in the range $gT\leqslant k\leqslant T$ does not contain the correct dependence on the momentum transfer. So the result would be wrong by a factor of two. [The correct spectral density in that range is proportional to $m_D^2\omega/(k^3(k^2-\omega^2))$.]

Briefly, concerning the (k,ω) integration in the vertex equation (9), this can be computed by retaining only the transverse part, giving the result

$$\int_{-k}^{k}\frac{d\omega}{2\pi}\frac{\rho_T}{\omega}\approx\frac{1}{k^2}. \tag{15}$$

This result is the same we would have obtained if, in the initial vertex equation (5), we had retained only the static mode in the sum with the substitution $D_{\alpha\beta}\to(\delta_{ij}-\hat{k}_i\hat{k}_j)1/k^2$. The following integration in k must be cut off by a semihard scale $\Lambda_{\max}\sim gT$ on one side and a soft scale $\Lambda_{\min}\sim g^2T$ on the other side. In addition, for a pair of fermionic external lines nearly on shell, for example $p^0\approx|\mathbf{p}|$, $p^0+q^0\approx|\mathbf{p}|$, we have only to retain in the vertex equation (9) one of the nearly singular products, $\Delta_+\Delta_+$ in that case. Of course, there is another pair of values of the external lines leading to a nearly singular piece, $p^0\approx-|\mathbf{p}|$, $p^0+q^0\approx-|\mathbf{p}|$, for which only the product $\Delta_-\Delta_-$ must be retained. This additional simplification is due to the fact that when $k\ll p$, a small momentum transfer cannot change the mass shell of the hard particles involved in the scattering.

Now, we make the following ansatz for the effective vertex[1]

$$\Gamma^{i,a}(p+q,q)=(A\gamma^i+B\gamma^0\hat{p}^i+C\gamma^j\hat{p}^i\hat{p}^j)t^a, \tag{16}$$

where A,B,C are some scalar functions which can depend on p^0,q^0 and $|\mathbf{p}|$. First we consider the term with $\Delta_+\Delta_+$. The equation for the effective vertex is now

$$\Gamma^{i,a}(p+q,q)=\Gamma_0^{i,a}-\frac{g^2Tt^a}{2N_c}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Delta_+(p+k+q)\Delta_+ \times(p+k)(A+B+C)(\gamma^0-\vec{\gamma}\cdot\hat{\mathbf{k}}(\widehat{\mathbf{p}+\mathbf{k}})\cdot\hat{\mathbf{k}}) \times(\widehat{p+k})^i\frac{1}{|\mathbf{k}|^2}. \tag{17}$$

The next step is to compute the integrals over the momentum transfer $\mathbf{k}$. It is useful to introduce a momentum variable u, defined as $u^2=(\mathbf{k}+\mathbf{p})^2$, so that

$$\int\frac{d^3\mathbf{k}}{(2\pi)^3}\ldots\to\frac{1}{(2\pi)^2p}\int_{\Lambda_{\min}}^{\Lambda_{\max}}dk\,k\int_{|k-p|}^{k+p}du\,u\cdots \tag{18}$$

One may decouple the limits of integration in the following way. In the limit $\mathbf{q}=0$, $q^0\to0$, with the pair of external fermion lines nearly on-shell at hard momentum, $p^0\sim|\mathbf{p}|\sim T$, we are concerned with a pair of internal fermionic lines sharing nearly the same loop momenta, so we can anticipate that values of $u\sim p^0$ and $u\sim p^0+q^0$ are important in the integration process. Since most of the contribution is in the region $u\sim p^0$, the limits on the integral over u can be extended to $\pm\infty$, with the contribution outside the initial range being small. Furthermore, if the scalar functions A,B,C are not strongly dependent on p we can take them out of the integral, along with the factor u, evaluated in $u=|\mathbf{p}|$. Given these approximations, the u integration results in

$$\int_{-\infty}^{\infty}du\frac{1}{i\omega_p-u+i\gamma_f\,\mathrm{sgn}(\omega_p)} \times\frac{1}{i\omega_p+i\nu_q-u+i\gamma_f\,\mathrm{sgn}(\omega_p+\nu_q)} =\frac{2\pi\mathcal{D}(\omega_p,\nu_q)}{|\nu_q|+2\gamma_f}, \tag{19}$$

[1]The possible structure of the vertex is greatly simplified by the fact that $\mathbf{q}=0$.

where $\mathcal{D}(\omega_p,\nu_q)=\theta(-\omega_p)\theta(\omega_p+\nu_q)+\theta(\omega_p)\theta(-\omega_p-\nu_q)$. We see that the thermal width regulates a singular behavior when ν_q vanishes. Upon completion of all these integrations, it is straightforward to solve for the scalar functions. Finally, we obtain for the vertex function when the external fermions are nearly on shell at leading logarithmic order

$$\Gamma^{ia}(\omega_p+\nu_q,\omega_p;\mathbf{p})=\gamma^it^a-\frac{2\gamma_f\mathcal{D}(\omega_p,\nu_q)}{(N_c^2-1)|\nu_q|+2N_c^2\gamma_f}\gamma^0\hat{p}^it^a, \tag{20}$$

where the logarithmic dependence on g coming from the k integral has been replaced by the same logarithm from the fermion damping rate. This result is consistent with the assumption about the smooth dependence on $|\mathbf{p}|$ of the scalar functions of the vertex. If we consider the term $\Delta_-\Delta_-$ we arrive at the same result. Here it should be emphasized that the vertex correction arising from ladder summation is of the same order as the one at tree level.

It remains to compute the polarization tensor

$$\Pi_{ab}^{ij}(\nu_q,\mathbf{0})=g^2N_fT\sum_{\omega_p}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mathrm{tr}[\Gamma^{i\,a}(\omega_p+\nu_q,\omega_p;\mathbf{p}) \times S(\omega_p,\mathbf{p})\gamma^jt^bS(\omega_p+\nu_q,\mathbf{p})]. \tag{21}$$

The most difficult task is to do the Matsubara sum. It will be useful to introduce a double spectral representation of the product of Green's functions as follows. Let $\Delta(p+q,p)=G(p+q)G(p)$, where the Green's function $G(p)$ has a single spectral representation

$$G(p)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\rho(\omega,\mathbf{p})}{p^0-\omega}. \tag{22}$$

Then, it is easy to check that $\Delta(p+q,p)$ admits the double spectral representation

$$\Delta(p+q,p)=\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\left[\frac{F_1(\omega_1,\omega_2)}{(p^0-\omega_1)(q^0-\omega_2)}+\frac{F_2(\omega_1,\omega_2)}{(p^0+q^0-\omega_1)(q^0-\omega_2)}\right], \tag{23}$$

where the corresponding F functions are

$$F_1(\omega_1,\omega_2)=\rho(\omega_1+\omega_2;\mathbf{p}+\mathbf{q})\rho(\omega_1;\mathbf{p}), \tag{24}$$

$$F_2(\omega_1,\omega_2)=-\rho(\omega_1;\mathbf{p}+\mathbf{q})\rho(\omega_1-\omega_2;\mathbf{p}). \tag{25}$$

Similarly, the product of the vertex correction and two propagators,

$$\delta\Delta(p+q,p)=\frac{\mathcal{D}(\omega_p,\nu_q)}{b+c|\nu_q|}\frac{1}{i\omega_p-|\mathbf{p}|+i\gamma\,\mathrm{sgn}(\omega_p)} \times\frac{1}{i\omega_p+i\nu_q-|\mathbf{p}|+i\gamma\,\mathrm{sgn}(\omega_p+\nu_q)}, \tag{26}$$

has the same representation with the δF functions

$$\delta F_1(\omega_1,\omega_2)=\frac{2b(\gamma^2+|\mathbf{p}|^2+\omega_1^2-2|\mathbf{p}|\omega_1-|\mathbf{p}|\omega_2+\omega_1\omega_2)-c\gamma\omega_2^2}{(\gamma^2+(\omega_1-|\mathbf{p}|)^2)(\gamma^2+(\omega_1-|\mathbf{p}|)^2-2|\mathbf{p}|\omega_2+2\omega_1\omega_2+\omega_2^2)(b^2+c^2\omega_2^2)}, \tag{27}$$

$$\delta F_2(\omega_1,\omega_2)=\frac{-2b(\gamma^2+|\mathbf{p}|^2+\omega_1^2-2|\mathbf{p}|\omega_1+|\mathbf{p}|\omega_2-\omega_1\omega_2)-c\gamma\omega_2^2}{(\gamma^2+(\omega_1-|\mathbf{p}|)^2)(\gamma^2+(\omega_1-|\mathbf{p}|)^2+2|\mathbf{p}|\omega_2-2\omega_1\omega_2+\omega_2^2)(b^2+c^2\omega_2^2)}. \tag{28}$$

At this point, we have all the elements required. There are two dominant contributions coming from the two products of internal propagators whose poles are nearly coincident, so in the following, we write twice the contribution corresponding to one of them [in particular, the one that has the singularity in $p^0=|\mathbf{p}|$, see Eqs. (31) and (32) below]. Making use of the double spectral representation to do the Matsubara sum and the fact that the angular dependence of the integrand is trivial, one obtains

$$\Pi_{ab}^{ij}(\nu_q,\mathbf{0})=\frac{2}{3}g^2N_f\delta^{ij}\delta_{ab}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega_2}{2\pi}\frac{1}{i\nu_q-\omega_2}\times\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}n_f(\omega_1)[F_1(\omega_1,\omega_2)+F_2(\omega_1,\omega_2)+\delta F_1(\omega_1,\omega_2)+\delta F_2(\omega_1,\omega_2)]. \tag{29}$$

Now, it easy to make the analytic continuation $i\nu_q\to q^0+i0^+$ and to expand the imaginary part to lowest order in q^0. The result is

$$\mathrm{Im}\,\Pi_{ab}^{Rij}(q^0,\mathbf{0})=-\frac{1}{3}g^2N_fq^0\delta^{ij}\delta_{ab}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}n_f'(\omega)\times[F_2(\omega,0)+\delta F_2(\omega,0)], \tag{30}$$

where we have used the property $(\delta)F_1(\omega_1,\omega_2)+(\delta)F_2(\omega_1+\omega_2,\omega_2)=0$. Thus, the explicit expressions of $F_2(\omega,0)$ and $\delta F_2(\omega,0)$ are required,

$$F_2(\omega,0)=-\rho_+(\omega,\mathbf{p})^2=-\frac{4\gamma_f^2}{((\omega-|\mathbf{p}|)^2+\gamma_f^2)^2}, \tag{31}$$

$$\delta F_2(\omega,0)=\frac{2}{N_c^2((\omega-|\mathbf{p}|)^2+\gamma_f^2)}, \tag{32}$$

which, in the limit $\gamma_f\to 0$, can be replaced by $F_2(\omega,0)=-2\pi\delta(\omega-|\mathbf{p}|)/\gamma_f$ and $\delta F_2(\omega,0)=2\pi\delta(\omega-|\mathbf{p}|)/(N_c^2\gamma_f)$. These substitutions give the final result to leading logarithmic order

$$\mathrm{Im}\,\Pi_{ab}^{Rij}(q^0,\mathbf{0})=-\frac{g^2T^2N_f}{36\gamma_f}q^0\delta^{ij}\delta_{ab}\left(1-\frac{1}{N_c^2}\right)=-q^0\delta^{ij}\delta_{ab}\frac{2\pi}{9}\frac{N_f}{N_c}\frac{T}{\log(1/g)}. \tag{33}$$

B. The gluonic contribution

We only need to consider transverse gluons propagating in the side rails of the ladder, since the longitudinal ones do not propagate at hard momentum. We again modify the gluon propagator in order to include the effects of a thermal width $2\gamma_g$

$$\rho(\omega,\mathbf{p})=\frac{1}{|\mathbf{p}|}\left(\frac{\gamma_g}{(\omega-|\mathbf{p}|)^2+\gamma_g^2}+\frac{\gamma_g}{(\omega+|\mathbf{p}|)^2+\gamma_g^2}\right), \tag{34}$$

where we use the gluon damping rate at hard momentum $\gamma_g=\alpha_sN_cT\ln(1/g)$. The vertex equation which sums the ladders is shown diagrammatically in Fig. 4. The normal vertex is

$$\Gamma_{0abc}^{ijk}(q,p,-p-q)=if^{abc}[2\delta^{jk}p^i-\delta^{ij}p^k-\delta^{ik}p^j]. \tag{35}$$

We parametrize the effective vertex as

$$\Gamma_{abc}^{ijk}(q,p,-p-q)=if^{abc}\left[Ap^i\mathcal{P}_{jk}^T(\hat{\mathbf{p}})+B\delta^{ij}p^k+C\delta^{ik}p^j+D\frac{p^ip^jp^k}{\mathbf{p}^2}\right]. \tag{36}$$

We need the effective vertex only when the indices j,k are contracted with the transverse projectors

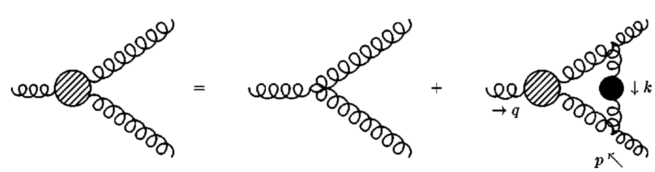

FIG. 4. Equation for the effective vertex which sums the gluon ladder contributions.

$$\Lambda^{ijk}_{abc}(q,p,-p-q)=\mathcal{P}^T_{jn}(\hat{\mathbf{p}})\Gamma^{inm}_{abc}(q,p,-p-q)\mathcal{P}^T_{mk}(\hat{\mathbf{p}})$$
$$=if^{abc}p^i\mathcal{P}^T_{jk}(\hat{\mathbf{p}})A. \tag{37}$$

After a bit of algebra the equation for the effective vertex reduces to

$$p^iA=2p^i+2g^2N_c\mathbf{p}^2T\sum_n\int\frac{d^3\mathbf{k}}{(2\pi)^3}A(p+k)^i$$
$$\times D^t(p+k+q)D^t(p+k)\frac{1}{|\mathbf{k}|^2}, \tag{38}$$

where $D^t(p)$ is the propagator obtained from the spectral density of Eq. (34). Within the same previous approximations, one obtains

$$\Lambda^{ijk}_{abc}(\omega_p+\nu_q,\omega_p;\mathbf{p})=2if^{abc}p^i\mathcal{P}^T_{jk}\left(1+\frac{\gamma_g\mathcal{D}(\omega_p,\nu_q)}{|\nu_q|+\gamma_g}\right). \tag{39}$$

With this result, following the same steps as earlier, it is easy to compute the imaginary part of the gluon polarization tensor. It reads

$$\mathrm{Im}\,\Pi^{Rij}_{ab}(q^0,\mathbf{0})=-\frac{g^2T^2N_c}{9\gamma_g}q^0\delta^{ij}\delta_{ab}. \tag{40}$$

Finally, the sum of both contributions gives the correct value for the color conductivity $\sigma_c=\omega_{\rm p}^2/\gamma_g$, where $\omega_{\rm p}^2\equiv(g^2T^2/18)(2N_c+N_f)$ is the plasma frequency.

III. SUMMARY

In this work we have derived the color conductivity to the leading logarithmic order. The main point here has been the evaluation of the Kubo formula for the current-current correlator within the framework of thermal field theory in the imaginary time formalism. We have shown how to do the summation of the ladder vertex corrections, making use of some approximations based on the kinematical regime of the scattering, which corresponds to exchange of quasistatic transverse gluons of soft momentum, $g^2T<|\mathbf{k}|<gT$. We have also shown how to introduce a double spectral representation of three-point functions in order to do the Matsubara sums involved in this formalism.

The possibility of computing other transport coefficients such as viscosity or electrical conductivity by a similar procedure deserves further consideration. This kind of transport properties follows from an integral over the differential cross section multiplied by the square of the momentum transfer k^2. The corrections due to Debye screening and Landau damping arising at order gT are sufficient to render this integral infrared finite, scaling as $g^4\log(k^*/m_{\rm el})\sim g^4\log(1/g)$, where k^* is a scale separating semihard and hard momentum transfers, restricted by $gT\ll k^*\ll T$ but otherwise arbitrary. So the $\log(1/g)$ comes from the sensitively to the momentum scale $m_{\rm el}\sim gT$, while the $\log(1/g)$ in the color conductivity and the damping rate comes from a sensitivity to the magnetic scale g^2T. This means that in order to attempt a similar computation of these transport coefficients, one must include into the rungs of the ladder the longitudinal part of interaction as well as the Landau damping effects. The infrared sensitivity to the scale g^2T should be compensated by a similar one coming from the hard damping rate.

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